Chapter 5

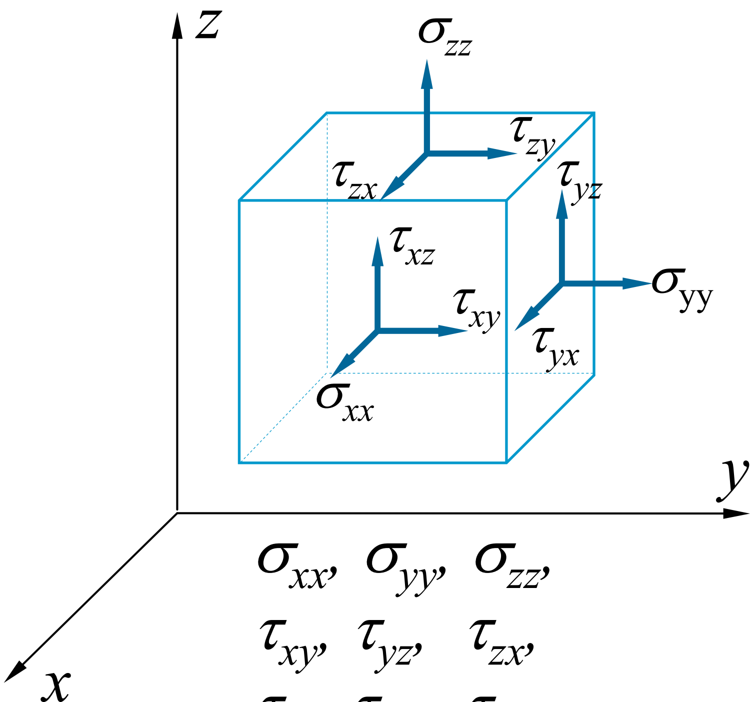
Dislocations-**2**

- Dislocation and Burgers vector
- Motion of the dislocation
- The stress field of dislocation
- The strain energy of dislocation
- Forces acted on dislocation
- Interactions between dislocations and defects
- Origin and proliferation of the dislocation

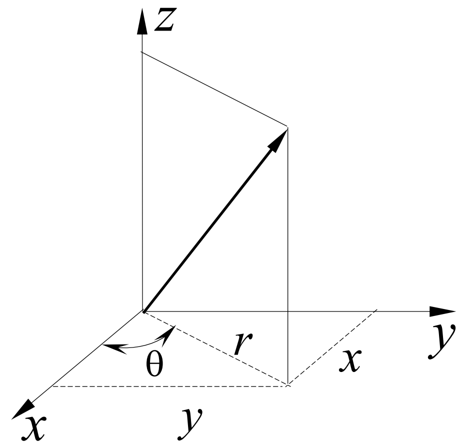
3. Stress field of the dislocation

[review]

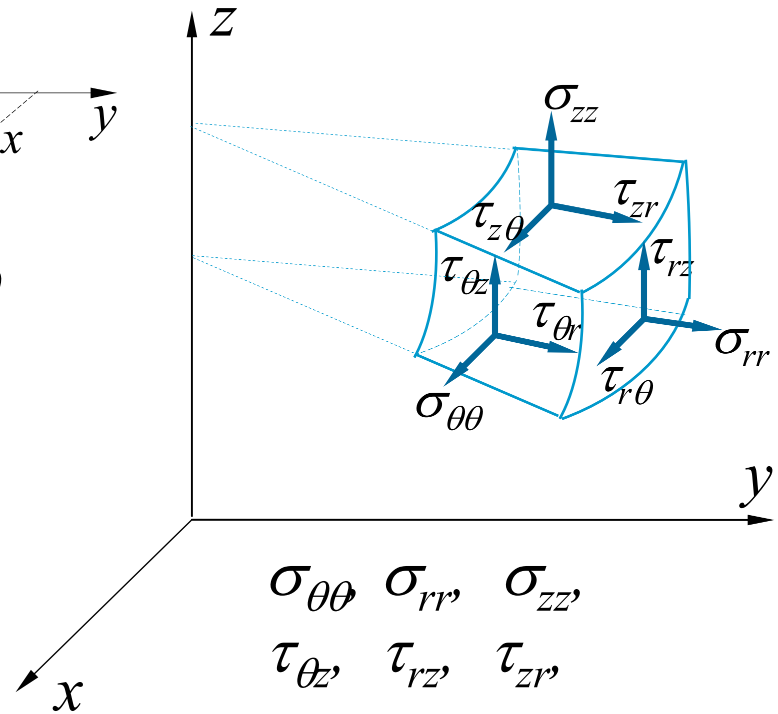
Stress' components



σ_{xx}, σ_{yy}, σ_{zz},
τ_{xy}, τ_{yz}, τ_{zx},
τ_{xz}, τ_{yx}, τ_{zy}



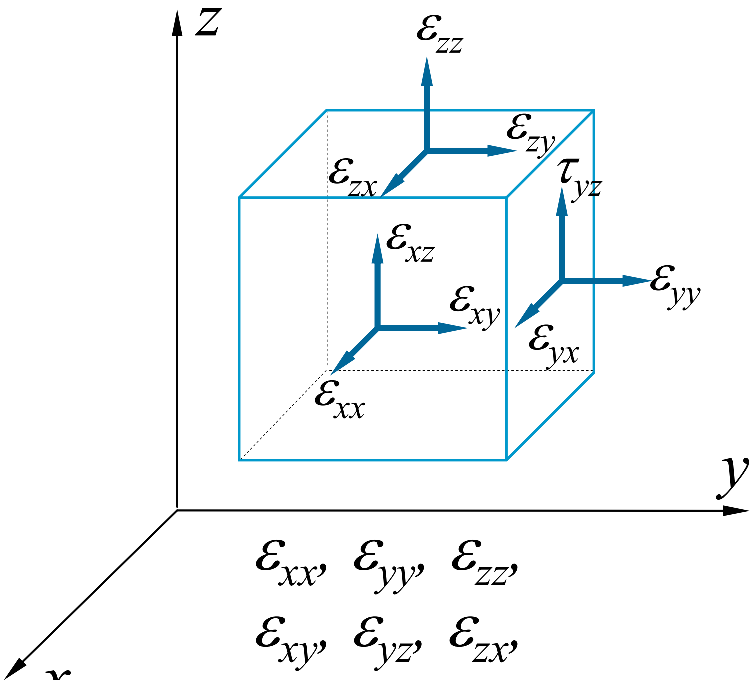
$$x = r\cdot\cos\theta$$
$$y = r\cdot\sin\theta$$
$$z = z$$



$\sigma_{\theta\theta}$, σ_{rr}, σ_{zz},
$\tau_{\theta z}$, τ_{rz}, τ_{zr},
$\tau_{\theta r}$, $\tau_{r\theta}$, $\tau_{z\theta}$

Strain components

Strain and Deformation



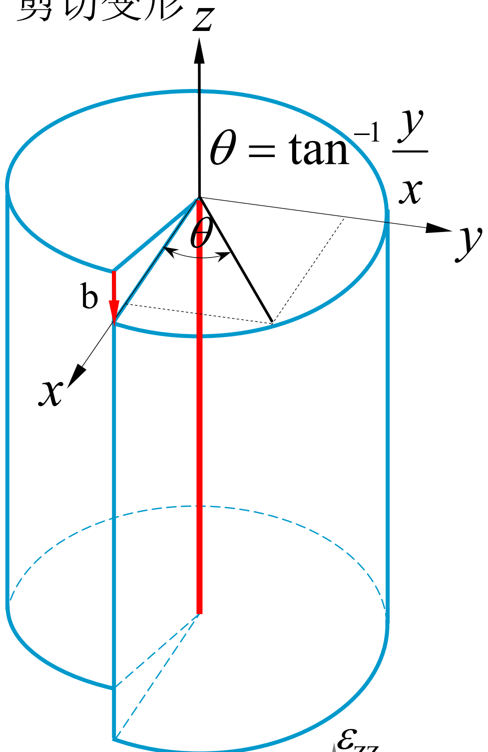
$\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{zz},$
$\varepsilon_{xy}, \varepsilon_{yz}, \varepsilon_{zx},$
$\varepsilon_{xz}, \varepsilon_{yx}, \varepsilon_{zy}$

deformation		u	v	w
		x	y	z
strain	$\varepsilon_{xx} =$	$\frac{du}{dx}$		
	$\varepsilon_{yy} =$		$\frac{dv}{dy}$	
	$\varepsilon_{zz} =$			$\frac{dw}{dz}$
	$\varepsilon_{xy} =$	$\frac{\partial u}{\partial y}$ +	$\frac{\partial v}{\partial x}$	
	$\varepsilon_{yz} =$		$\frac{\partial v}{\partial z}$ +	$\frac{\partial w}{\partial y}$
	$\varepsilon_{zx} =$	$\frac{\partial u}{\partial z}$	+	$\frac{\partial w}{\partial x}$

Screw dislocation

Shear deformation

剪切变形



elastic deformation

$$u = 0 \qquad v = 0 \qquad w = \frac{b}{2\pi}\theta = \frac{b}{2\pi}\tan^{-1}\frac{y}{x}$$

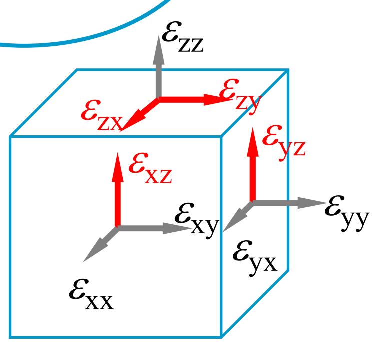
strains

$$\varepsilon_{zx} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} = \frac{b}{2\pi}\frac{\partial}{\partial x}\left(\tan^{-1}\frac{y}{x}\right) + 0 = \frac{-b \cdot y}{4\pi(x^2+y^2)}$$

$$\varepsilon_{zy} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} = \frac{b \cdot x}{4\pi(x^2+y^2)}$$

stresses

$$\tau_{zx} = \frac{-Gb \cdot y}{4\pi(x^2+y^2)} \qquad \tau_{zy} = \frac{Gb \cdot x}{4\pi(x^2+y^2)}$$



$$\varepsilon_{xx} = \varepsilon_{yy} = \varepsilon_{zz} = 0$$
$$\varepsilon_{yx} = \varepsilon_{xy} = 0$$
$$\varepsilon_{zx} = \varepsilon_{xz} \neq 0$$
$$\varepsilon_{zy} = \varepsilon_{yz} \neq 0$$

Hooke's Law

$$\tau = 2G\varepsilon$$

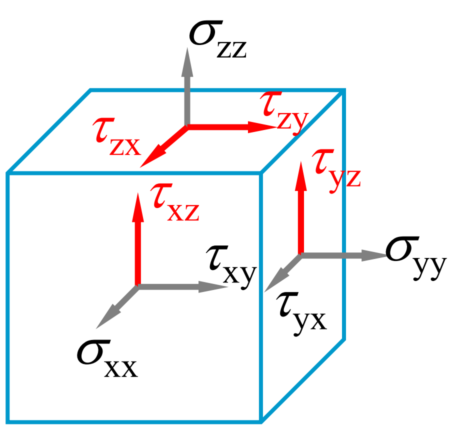
G is shear module

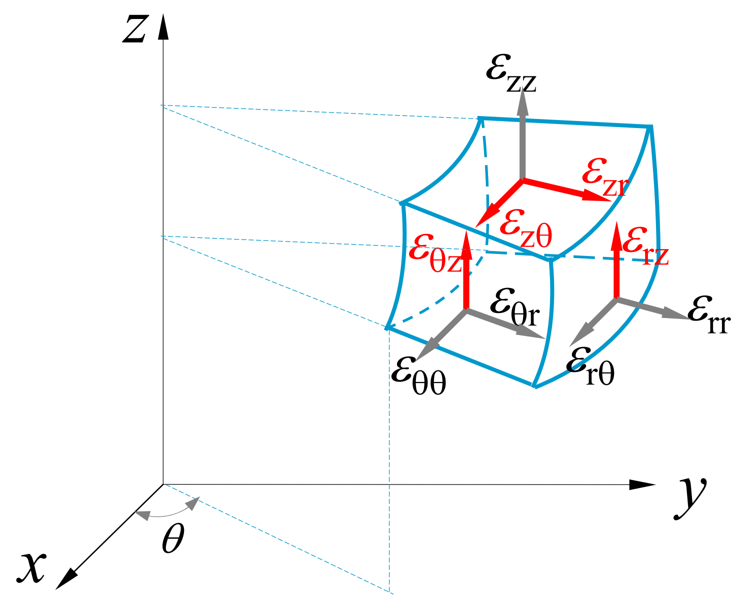
$$\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = 0$$
$$\tau_{yx} = \tau_{xy} = 0$$
$$\tau_{zx} = \tau_{xz} \neq 0$$
$$\tau_{zy} = \tau_{yz} \neq 0$$

Screw dislocation

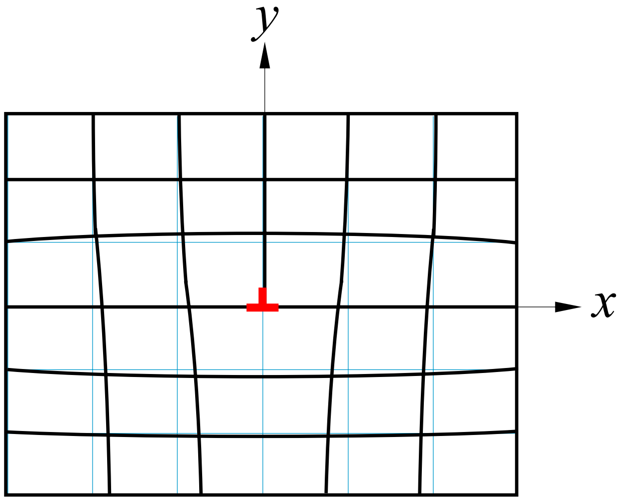
$$\sigma_{xx}=\sigma_{yy}=\sigma_{zz}=0$$
$$\tau_{yx}=\tau_{xy}=0$$
$$\tau_{zx}=\tau_{xz}=\frac{-Gb}{2\pi}\frac{y}{x^2+y^2}$$
$$\tau_{zy}=\tau_{yz}=\frac{Gb}{2\pi}\frac{x}{x^2+y^2}$$

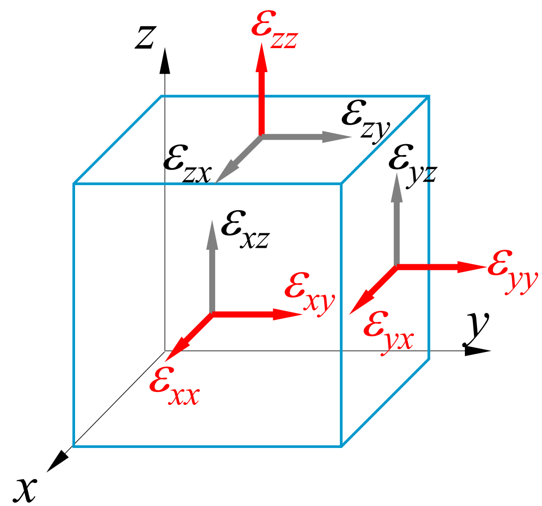
$$\sigma_{zz}=\sigma_{\theta\theta}=\sigma_{rr}=0$$
$$\tau_{\theta r}=\tau_{r\theta}=0$$
$$\tau_{\theta z}=\tau_{z\theta}=\tau_{rz}=\tau_{zr}=\frac{Gb}{2\pi r}$$





Edge dislocation





$$\varepsilon_{xz} = \varepsilon_{zx} = \varepsilon_{yz} = \varepsilon_{zy} = 0$$

ε_{xx} ε_{yy} ε_{zz}

ε_{xy} ε_{yx}

$$\tau_{xz} = \tau_{zx} = \tau_{yz} = \tau_{zy} = 0$$

σ_{xx} σ_{yy} σ_{zz}

τ_{xy} τ_{yx}

Elastic deformation

$$u = \frac{b}{2\pi}\left[\tan^{-1}\frac{y}{x} + \frac{1}{2(1-v)}\frac{xy}{x^2+y^2}\right]$$

$$v = \frac{b}{2\pi}\left[\frac{2v-1}{4(1-v)}\ln(x^2+y^2) - \frac{1}{(1-v)}\frac{x^2}{x^2+y^2}\right]$$

$$w = 0$$

Edge dislocation

$$\sigma_{xx} = \frac{-Gb}{2\pi(1-\nu)} \frac{y(3x^2+y^2)}{(x^2+y^2)^2}$$

$$\sigma_{yy} = \frac{Gb}{2\pi(1-\nu)} \frac{y(x^2+y^2)}{(x^2+y^2)^2}$$

$$\sigma_{zz} = \nu(\sigma_{xx}+\sigma_{yy})$$

$$\tau_{xy} = \tau_{yx} = \frac{Gb}{2\pi(1-\nu)} \frac{x(x^2-y^2)}{(x^2+y^2)^2}$$

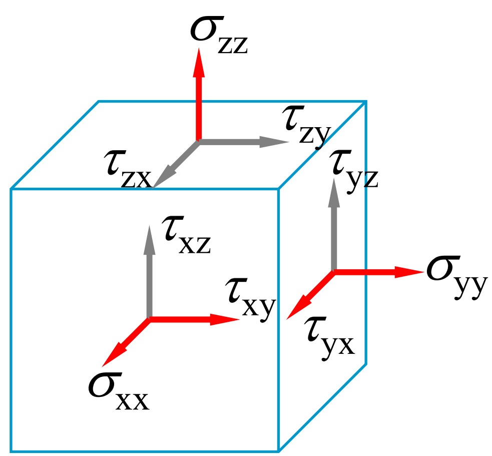
$$\tau_{xz} = \tau_{zx} = \tau_{yz} = \tau_{zy} = 0$$

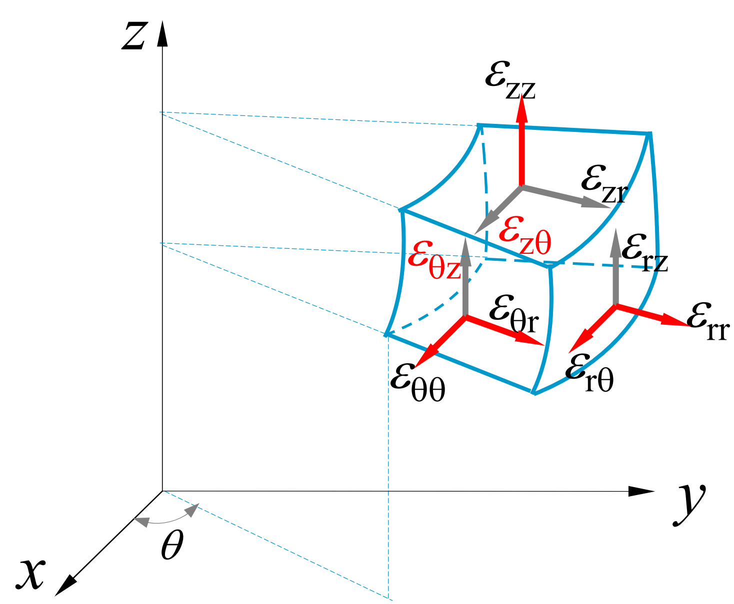
$$\sigma_{rr} = \sigma_{\theta\theta} = \frac{-Gb}{2\pi(1-\nu)} \frac{\sin\theta}{r}$$

$$\sigma_{zz} = -2\frac{Gb}{2\pi(1-\nu)} \frac{\nu\sin\theta}{r}$$

$$\tau_{r\theta} = \tau_{\theta r} = \frac{Gb}{2\pi(1-\nu)} \frac{\cos\theta}{r}$$

$$\tau_{z\theta} = \tau_{\theta z} = \tau_{rz} = \tau_{zr} = 0$$



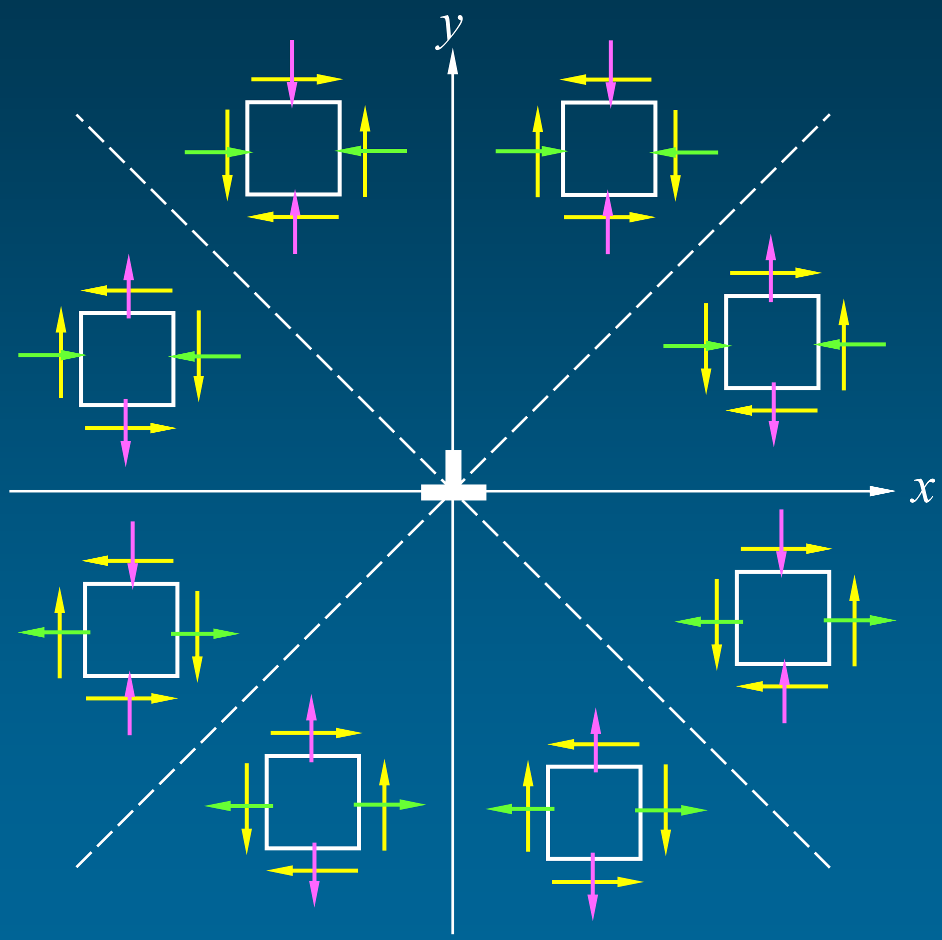


Edge dislocation

$$\sigma_{xx} = \frac{-Gb}{2\pi(1-\nu)} \frac{y(3x^2+y^2)}{(x^2+y^2)^2}$$

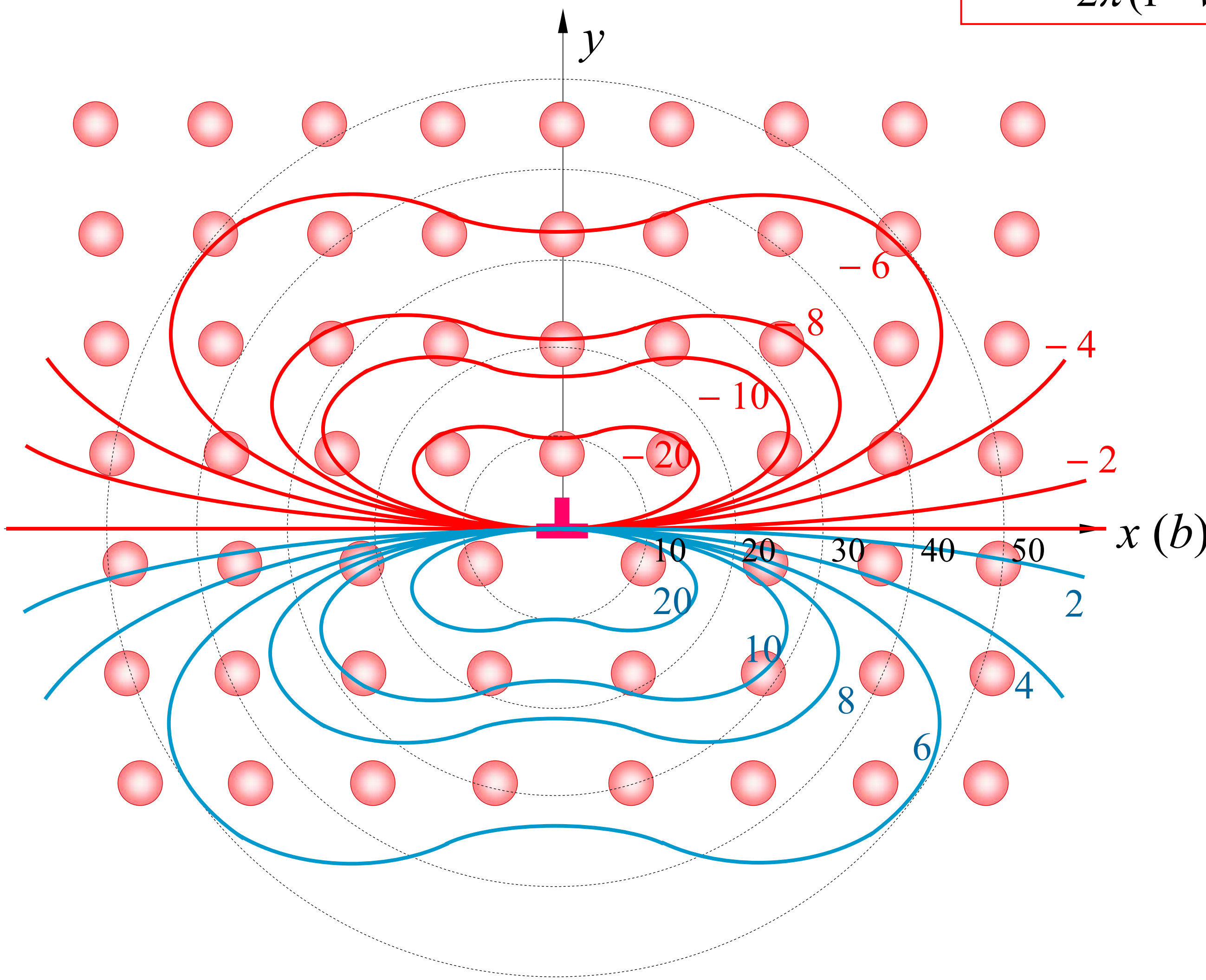
$$\sigma_{yy} = \frac{Gb}{2\pi(1-\nu)} \frac{y(x^2-y^2)}{(x^2+y^2)^2}$$

$$\tau_{xy} = \tau_{yx} = \frac{Gb}{2\pi(1-\nu)} \frac{x(x^2-y^2)}{(x^2+y^2)^2}$$



Edge dislocation

$$\sigma_{xx} = \frac{-Gb}{2\pi(1-v)} \frac{y(3x^2+y^2)}{(x^2+y^2)^2}$$



Stress field of the σ_{xx} of the edge dislocation at *xoy* plane.

Section 4.3 Dislocations-2

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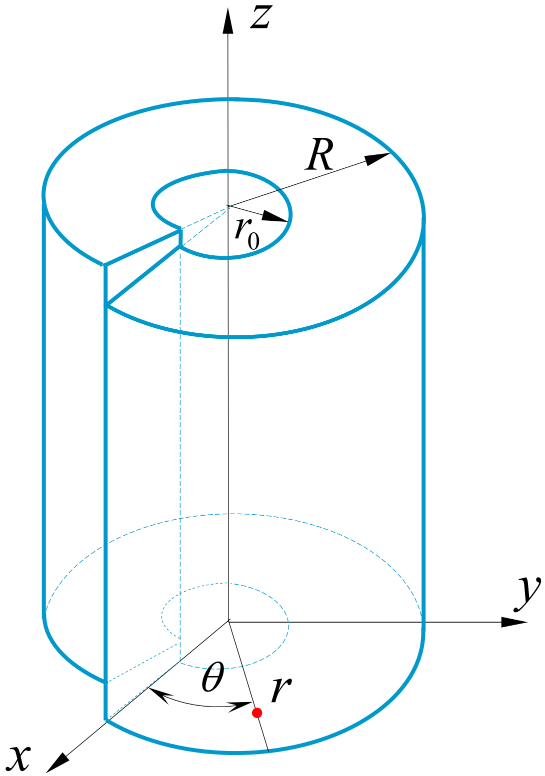
4. Strain energy of the dislocation (*E*)

$$E = E_c + E_s$$

~10%

Screw dislocation



Work required to create a dislocation

$$W_s = E_s$$

Strain energy of the dislocation

Strain

$$\tau_{z\theta} = \frac{Gb}{2\pi \cdot r}$$

Hooke's law: $w_s = \frac{\tau_{z\theta}}{2} bdr = \frac{Gb}{4\pi \cdot r} bdr = \frac{Gb^2}{4\pi \cdot r} dr$

$$W_S = \int_{r_0}^{R} w_s = \int_{r_0}^{R} \frac{Gb^2}{4\pi \cdot r} dr = \frac{Gb^2}{4\pi \cdot r} \ln \frac{R}{r_0}$$

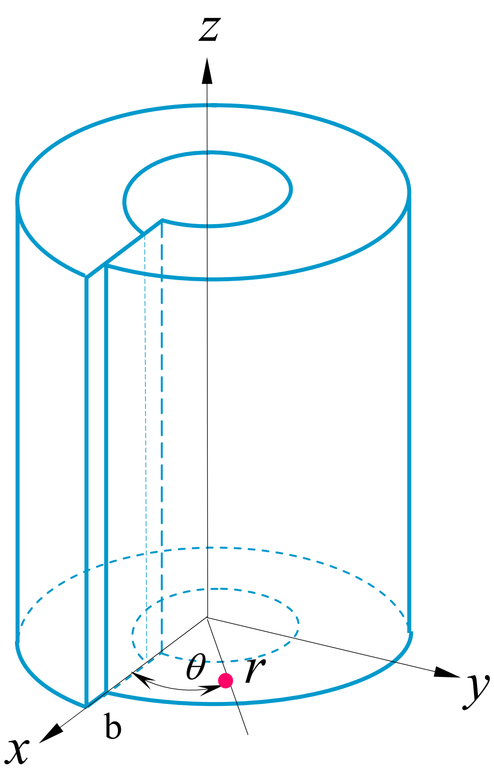
$$E_S = \frac{Gb^2}{4\pi} \ln \frac{R}{r_0}$$

Strain energy of dislocation per unit length

Edge dislocation

The dislocation is on the slip plane, *i.e.*, $\theta = 0$

It is, therefore, found that $\cos\theta = 1$



$$\tau_{r\theta} = \frac{Gb}{2\pi r(1-\nu)}\frac{\cos\theta}{r} = \frac{Gb}{2\pi r(1-\nu)}\frac{1}{r}$$

$$E_e = \frac{1}{2}\int_{r_0}^{R}\tau_{r\theta} b\,dr = \frac{1}{2}\int_{r_0}^{R}\frac{Gb^2}{4\pi(1-\nu)} b\frac{dr}{r}$$

$$= \frac{Gb^2}{4\pi r(1-\nu)}\ln\frac{R}{r_0}$$

$$E_e = \frac{Gb^2}{4\pi(1-\nu)}\ln\frac{R}{r_0}$$

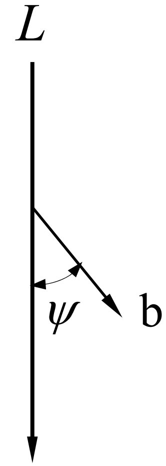
Summary of the strain energy of dislocation

Mixed dislocation

$$E_m = E_s + E_e$$

$$E_s = \frac{Gb_s^2}{4\pi}\ln\frac{R}{r_0}$$

$$E_e = \frac{Gb_e^2}{4\pi(1-\nu)}\ln\frac{R}{r_0}$$



$$b \begin{cases} b_s = b\cdot\cos\psi \\ b_e = b\cdot\sin\psi \end{cases}$$

$\psi = 90^\circ$, edge dislocation

$\psi = 0^\circ$, screw dislocation

$$E_m = \frac{Gb^2\cos^2\psi}{4\pi}\ln\frac{R}{r_0} + \frac{Gb^2\sin^2\psi}{4\pi(1-\nu)}\ln\frac{R}{r_0}$$

$$= \frac{Gb^2}{4\pi(1-\nu)}\ln\frac{R}{r_0}(1-\nu\cos^2\psi)$$

Example, for copper (**Cu**), $G = 4 \times 10^6$ N/m², b = 2.5×10^{-8} cm. It is found that $E_m \sim 2.5 \times 10^{-11}$ J/cm².

- E is proportional to $|b|^2$, $E \propto Gb^2$
- $E_s \sim 0.6Gb^2$, $E_e \sim 0.9Gb^2$, $E_m \sim 0.6\text{-}0.9Gb^2$
- curve → straight

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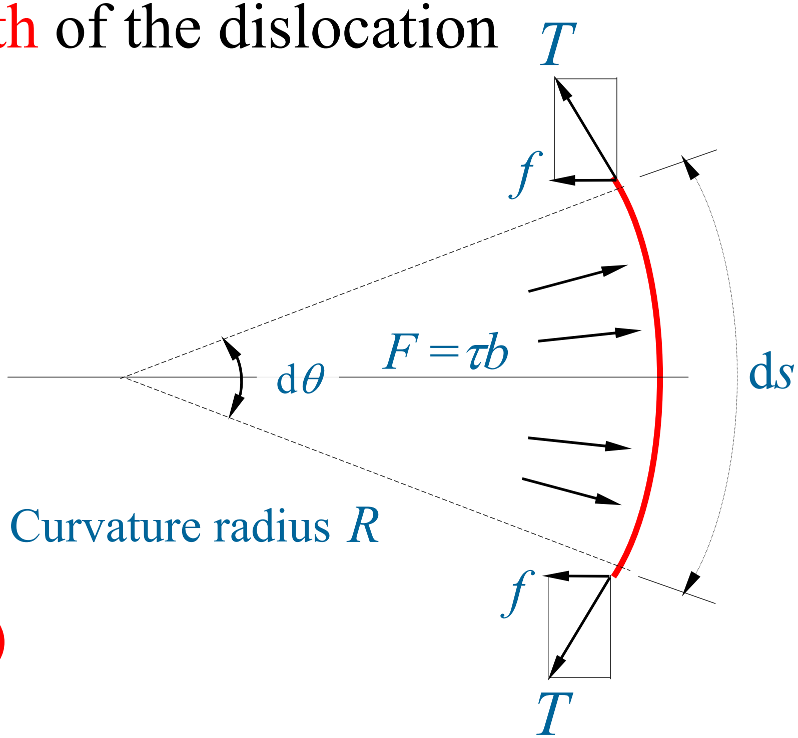
5. Forces acted on the dislocation

5-1. Linear tension of the dislocation (T)

Energy needed for increasing an unit length of the dislocation (strain energy, E).



$$T = E = \alpha G b^2 \quad \begin{cases} \alpha = 1, \text{ when it is straight} \\ \alpha = 0.5, \text{ when it is curve} \end{cases}$$

$$f = 2[T \cdot \sin(\mathrm{d}\theta/2)] \approx T \cdot \mathrm{d}\theta = T \cdot \mathrm{d}s/R \quad (1)$$

The force, F, acting on an unit length of the dislocation by external shear stress (τ) is given that

$$F = \tau \cdot b$$

The total force ($\mathcal{F}$) on the curve dislocation is, therefore, given by that

$$\mathcal{F} = F \cdot \mathrm{d}s \qquad (2)$$

Linear tension of the dislocation

$$f = T \cdot ds/R \qquad (1)$$

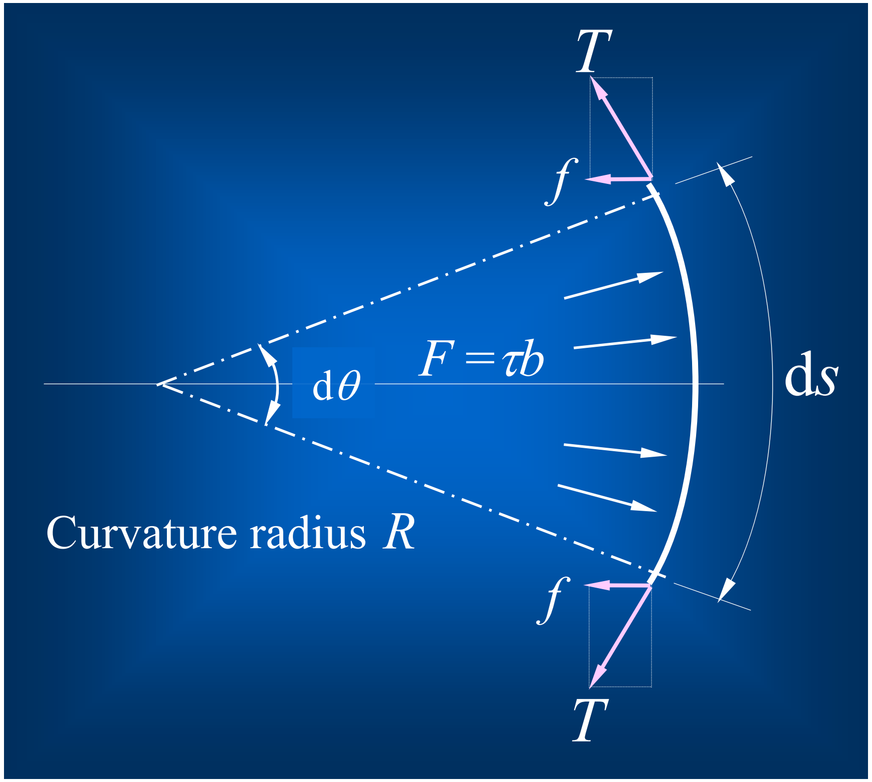
Force acted on the dislocation

$$\mathcal{F} = F \cdot ds \qquad (2)$$

In equilibrium, one finds that

$$f = \mathcal{F}$$

$$T \cdot ds/R = F \cdot ds \Rightarrow \mathbf{F = T/R}$$



*The **force** acted on a dislocation is proportional to its linear tension and is inversely proportional to the curvature radius of the dislocation.*

$$F = \tau \cdot b$$

The correlation between the curvature radius of the dislocation, *R*, and an external shear stress, τ.

$$F = T/\mathrm{R} \qquad T = \alpha G b^2$$

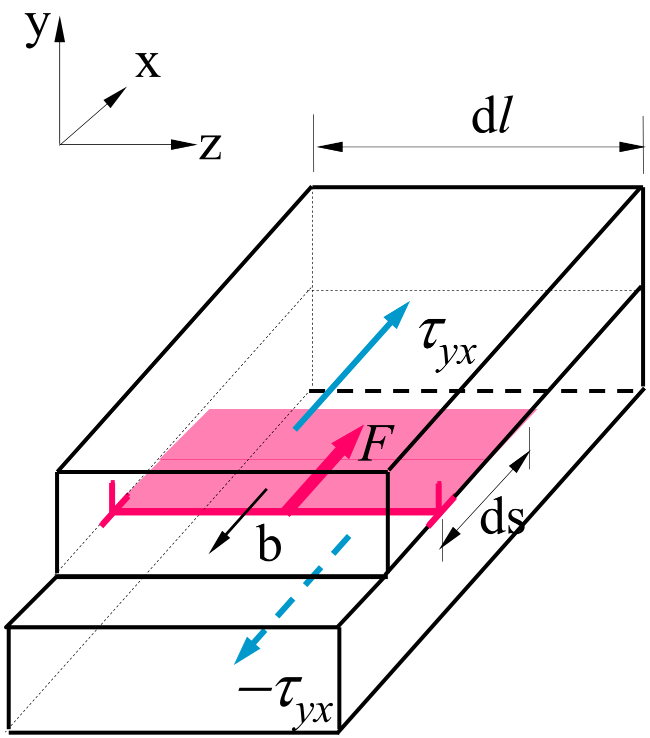
$$F = \alpha G b^2/R \qquad F = \tau b$$

$$\tau = \alpha G b/R$$

$$\tau \sim G b/2R$$

5.2 Force applied on the dislocation in stress field

Force that causes the dislocation to move, $\boldsymbol{F}$.



done by τ_{xy}

$$dW_1 = \boldsymbol{\tau}_{yx} \cdot dA \cdot \boldsymbol{b}$$

$$dW_2 = F \cdot dl \cdot ds$$

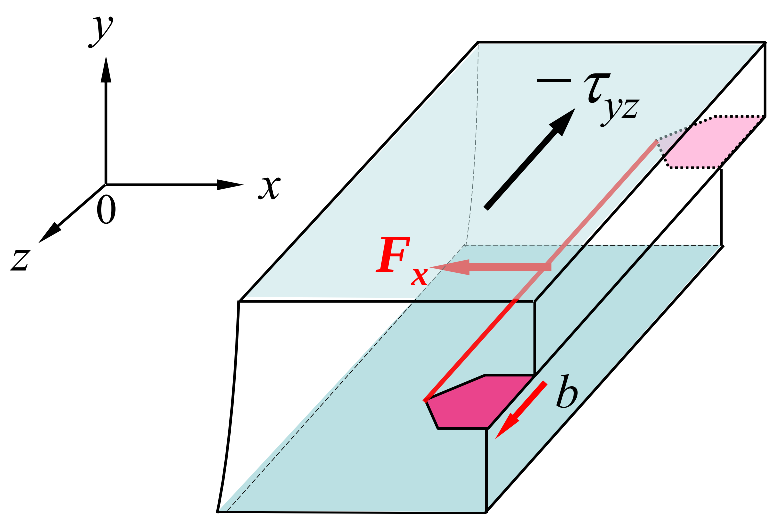
$$dW_1 = dW_2$$

$$F = \tau_{yx} \cdot b$$

Slip drive

Force that causes the dislocation slip

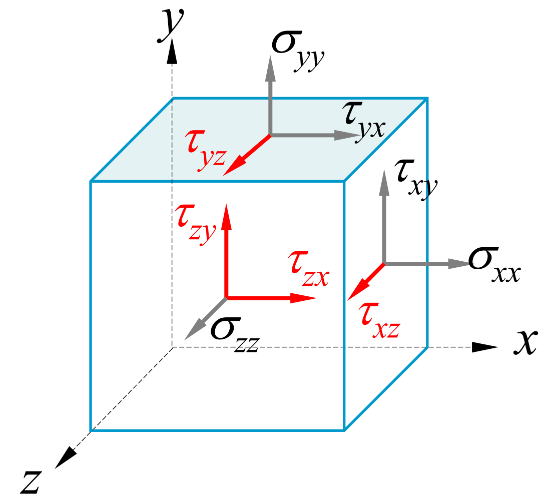
Screw dislocation

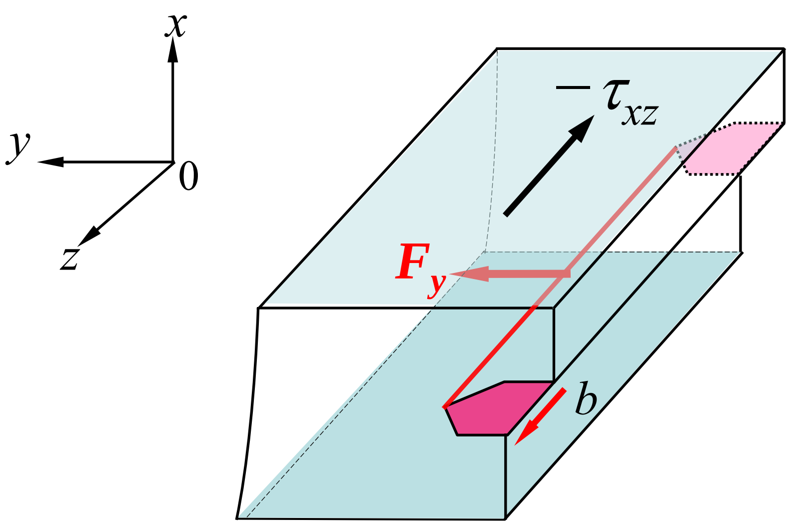


Slip drive, F_x

$$-\boldsymbol{F}_{\boldsymbol{x}} = -\tau_{yz} \cdot \boldsymbol{b}$$

$$\boldsymbol{F}_{\boldsymbol{x}} = \tau_{yz} \cdot \boldsymbol{b}$$



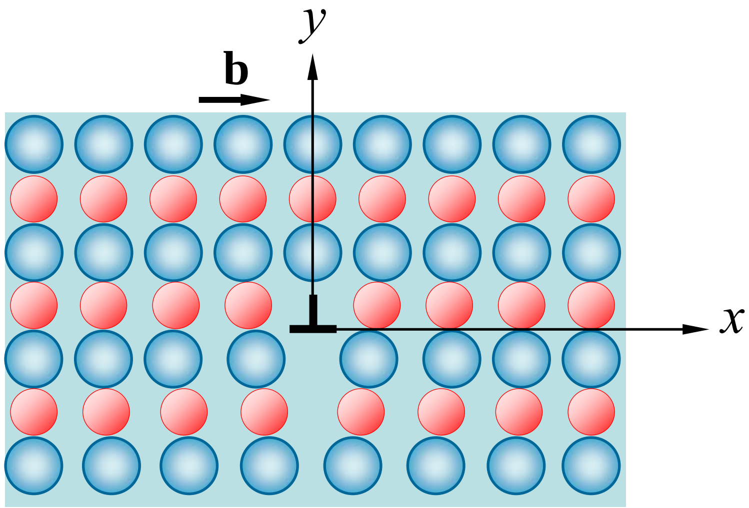


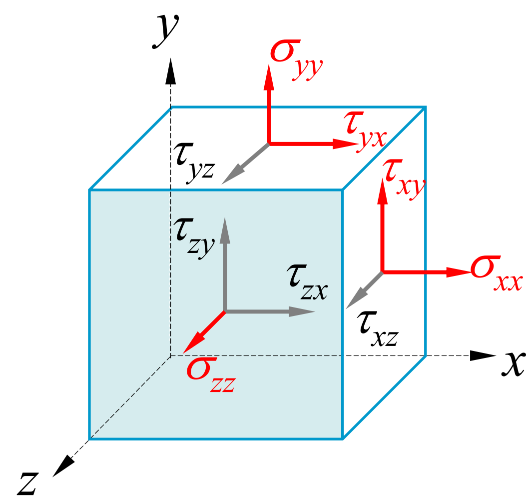
Slip drive, F_y

$$\boldsymbol{F}_{\boldsymbol{y}} = -\tau_{xz} \cdot \boldsymbol{b}$$

$$\boldsymbol{F}_{\boldsymbol{y}} = -\tau_{xz} \cdot \boldsymbol{b}$$

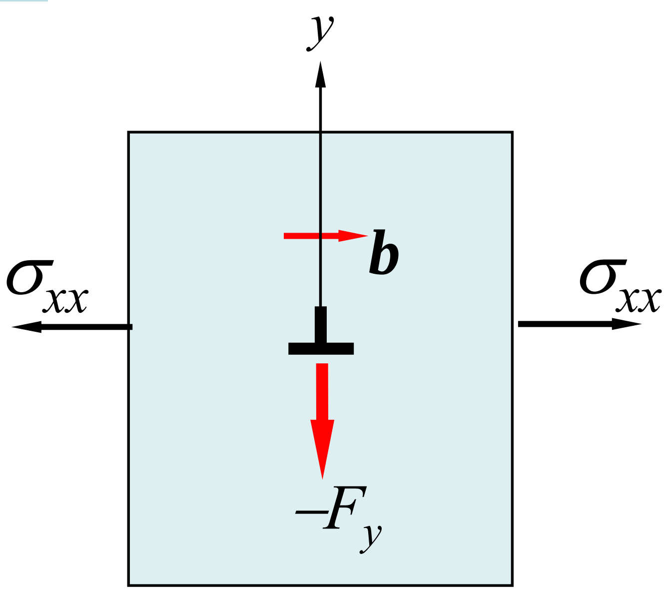
Force that causes the dislocation climb



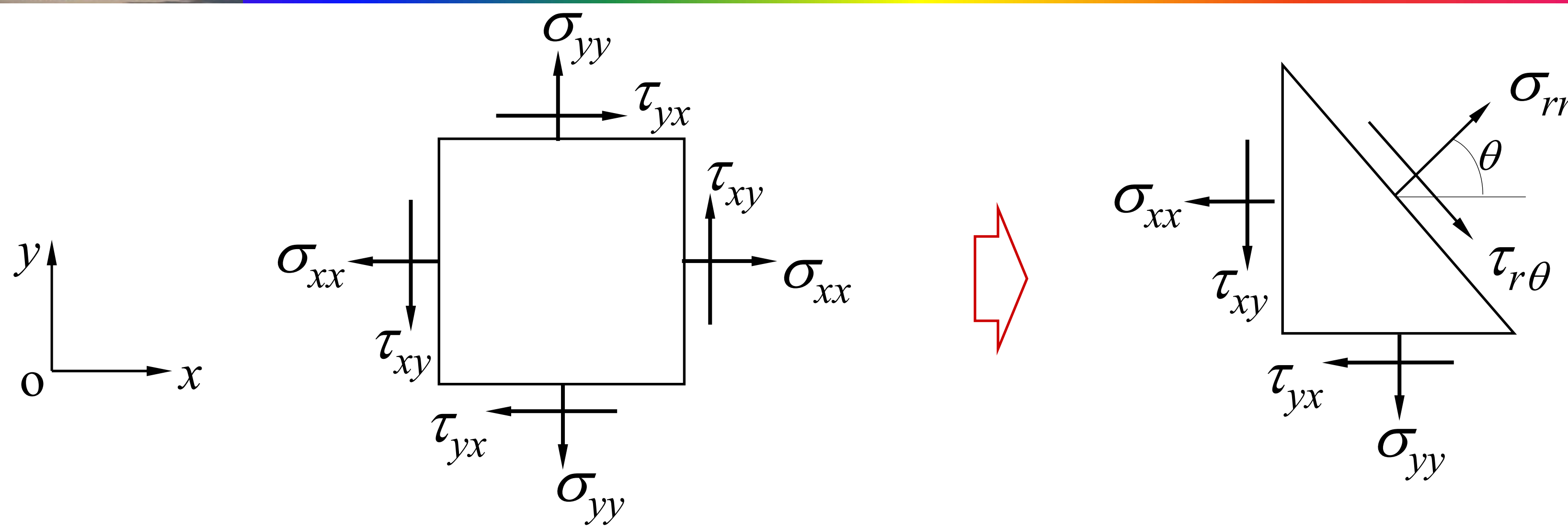


$$F_y = -\sigma_{xx} \cdot b$$

Climb drive



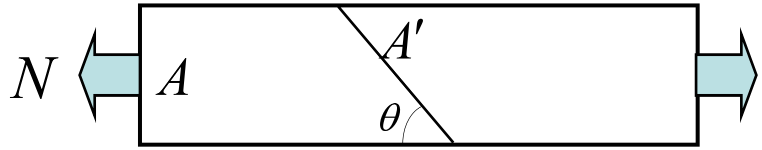
motion drive



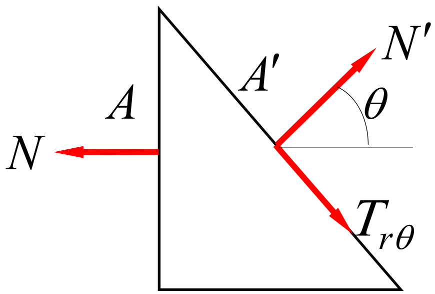
$$\sigma_{rr} = \sigma_{xx}\cos^2\theta + \sigma_{yy}\sin^2\theta + \tau_{xy}\sin\theta\cos\theta$$

$$\cos\theta = \frac{x}{\sqrt{x^2 + y^2}} \qquad \sin\theta = \frac{y}{\sqrt{x^2 + y^2}}$$

$$\sigma_{rr} = \frac{1}{x^2 + y^2}(y^2\sigma_{xx} + x^2\sigma_{yy} + xy\tau_{xy})$$



$$\sigma_{xx} = \frac{N}{A}$$

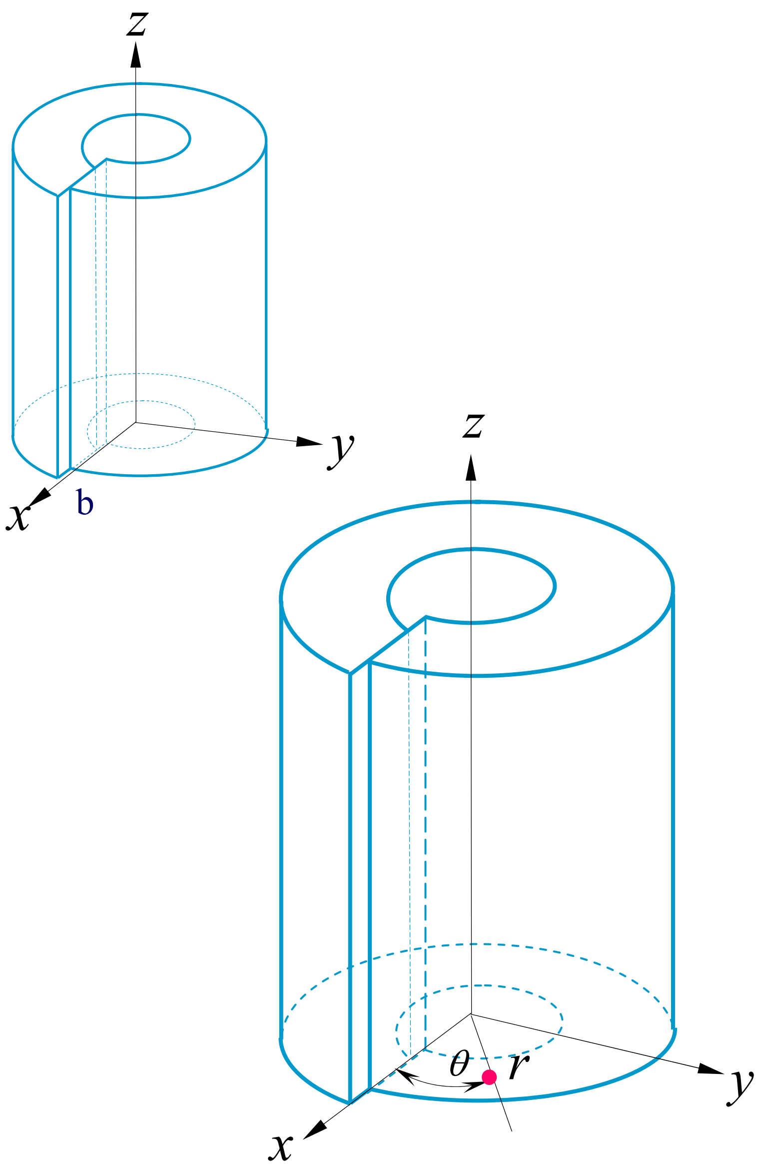


$$N\cos\theta$$

$$\sigma_{xx} = \frac{N}{A} \qquad \sigma_{rr} = \frac{N'}{A'} = \frac{N}{A}\cos^2\theta = \sigma_{xx}\cos^2\theta$$

$$\frac{A}{\cos\theta}$$

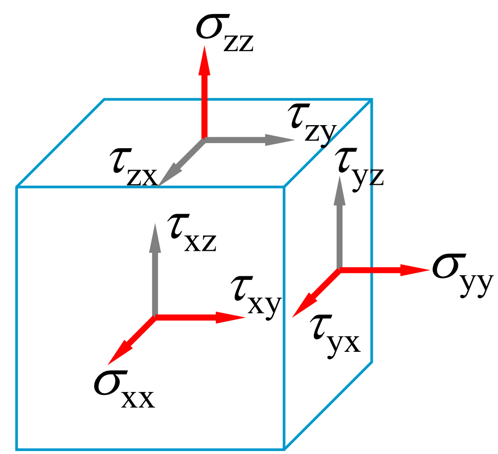
Edge dislocation



$$\tau_{xz} = \tau_{zx} = \tau_{yz} = \tau_{zy} = 0$$

$$\sigma_{xx} \quad \sigma_{yy} \quad \sigma_{zz}$$

$$\tau_{xy} \quad \tau_{yx}$$

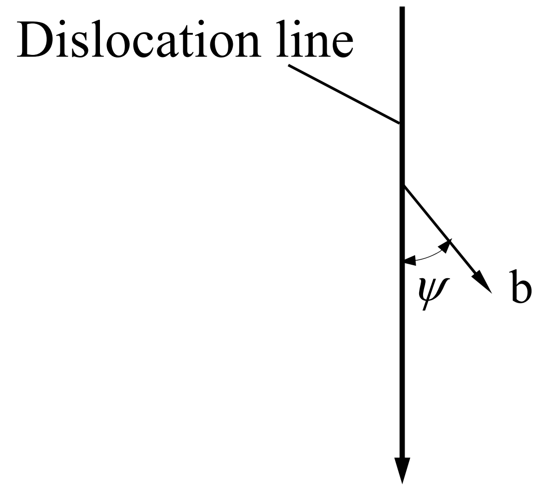


Summary of the strain energy of dislocation

$$E_s = \frac{Gb^2}{4\pi} \ln \frac{R}{r_0}$$

$$E_e = \frac{Gb^2}{4\pi(1-\nu)} \ln \frac{R}{r_0}$$

$$E_m = \frac{Gb^2}{4\pi(1-\nu)} \ln \frac{R}{r_0} (1-\nu\cos^2\psi)$$



- E is proportional to $|b|^2$, $E \propto Gb^2$
- $E_s \sim 0.6Gb^2$, $E_e \sim 0.9Gb^2$, $E_m \sim 0.6\text{-}0.9Gb^2$
- curve → straight